The Study of Shocks in Three-States Driven-Diffusive Systems: A Matrix Product Approach

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Abstract

We study the shock structures in three-states one-dimensional driven-diffusive systems with nearest neighbors interactions using a matrix product formalism. We consider the cases in which the stationary probability distribution function of the system can be written in terms of superposition of product shock measures. We show that only three families of three-states systems have this property. In each case the shock performs a random walk provided that some constraints are fulfilled. We calculate the diffusion coefficient and drift velocity of shock for each family.

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1 Introduction

During recent years, much attention has been paid to the study of out of equilibrium low-dimensional diffusive particles systems [1, 2]. These systems are kept far from equilibrium by maintaining a steady state particle current. This results in various critical phenomena including boundary-induced phase transitions, phase separation and spontaneous symmetry breaking. For instance the evolution of shocks in these systems is an indication for the phase separation phenomenon. Macroscopically these shocks are special solutions of nonlinear hydrodynamic equations for coarse-grained quantities like the density of particles. Nevertheless in order to understand the structure of the shocks and also the nonlinear nature of them it is necessary to study the microscopic dynamics of these systems in details.

A simple two-states lattice gas model, which shows a variety of interesting critical phenomena including shock formations, is the Partially Asymmetric Simple Exclusion Process (PASEP) with open boundaries. In the PASEP the particles are injected into the system and also extracted from the boundaries while hop in the bulk of a discrete lattice [3]. Recent investigations show that for the PASEP

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with open boundaries (and even on an infinite lattice) a traveling shock with a step-like density profile might evolve in the system provided that microscopic reaction rates are adjusted appropriately [4, 5, 6, 7, 8, 9]. The shock position in this case performs a random walk while reflects from the boundaries. It is also known that for the PASEP multiple shocks can evolve in the system provided that specific constraints are fulfilled. It turns out that the PASEP is not the only two-states lattice gas model in which the dynamics of the single shock under the Hamiltonian of the system is a random walk. There are also two other systems of this kind i.e. the Branching-Coalescing Random Walk (BCRW) and also the Asymmetric Kawasaki-Glauber Process (AKGP). The steady state probability distribution functions of these three systems are known to be made of superposition of product shock measures [9].

In the context of one-dimensional out of equilibrium reaction-diffusion systems, the Matrix Product Formalism (MPF) has shown to be one of powerful technics in order to study of both the steady state and also the dynamical properties of these systems [2]. According to this approach the probability distribution function of the system at any time (including the steady state) can be written in terms of the matrix element (for open boundary conditions) or trace (for periodic boundary conditions) of products of some non-commuting operators. At large times these operators are time independent while they can be time dependent at finite times. For the systems with nearest neighbors interactions these operators satisfy an algebra which can have finite or infinite-dimensional representations. For the three above mentioned systems, it has been shown that the conditions for the existence of a single traveling shock with random walk behavior in the system are exactly those for the existence of a two-dimensional representation for their quadratic algebras [10]. On the other hand, it is well known that the quadratic algebra of the PASEP has also an n-dimensional representation given that some extra constraints on the boundaries and bulk rates are held [11, 12]. It turns out that these constraints are exactly those for the existence of $n-1$ consecutive shocks with random walk dynamics in the system. In this case the steady state probability distribution function of the PASEP can be expressed in terms of superposition of $n-1$ shocks.

In this paper we consider the most general three-states reaction-diffusion system with nearest neighbors interactions and open boundaries. The system is defined on a lattice of length L. Throughout this paper, we assign the letters A, B and E to these states. The E's are associated with the empty sites or holes in the system while A's and B's are associated with particles of different types. We define the total density of particles as the sum of the densities of particles of type A and B at each site. We also assume that the density of A particles in a given site is proportional to the density of B particles at the same site. In terms of the total density of the particles and the density of vacancies, the three-states system is basically a two-states system. In present work, we firstly investigate the conditions under which the stationary probability distribution function of the three-states system can be expressed in terms of superposition of single shocks. Our approach will be based on the MPF. From the MPF point of view, one can simply try to map the resultant algebra of the three-states system into that of three two-states above mentioned models and find the constraints on the microscopic reaction rates under which this mapping is possible. This guarantees that the resultant algebra of the three-states system has a two-dimensional representation which in turns indicates that the steady state distribution of the

system can be written in terms of superposition of single shocks. In the case of mapping the quadratic algebra of the three-states system into the PASEP's algebra, the constraints are sufficient for expressing the stationary probability distribution of the system in terms of superposition of multiple shocks.
Secondly we study the dynamics of a single product shock measures in the three-states systems. The dynamics of a single product shock measure in two-states systems has already been studied and as we mentioned in the PASEP, the BCRW and the AKGP the shock performs a random walk given that some constraints are satisfied. For our three-states system the conditions under which the quadratic algebra of the system has a two-dimensional representation are exactly those for the product shock measure to have a random walk dynamics. We obtain the diffusion coefficient and the shock drift velocity in each case.
This paper is organized as follows: In section 2 we briefly review the basic concepts of the MPF especially for two and three states system. The quadratic algebras of the PASEP, the BCRW and also the AKGP are reviewed in section 3. We also review the constraints under which their quadratic algebras have two-dimensional representations. In section 4 we map the quadratic algebra of the most general three-states system (in terms of the total density of the particles and the vacancies) to the quadratic algebras of these three models and find the constraints under which the mapping is possible. The dynamics of the shock is discussed in section 5. The results and conclusion are brought in the last section.

2 The Matrix Product Formalism (MPF)

In this section we review the basic concepts of the MPF introduced in [3]. Let us define $P(\mathcal{C};t)$ as the probability distribution function of any configuration $\mathcal{C}$ of a Markovian interacting particle system at the time t. The time evolution of $P(\mathcal{C};t)$ can be written as a Schrödinger equation in imaginary time

$$\frac{d}{dt}P(\mathcal{C};t) = HP(\mathcal{C};t) \tag{1}$$

in which H is a stochastic Hamiltonian. The matrix elements of the Hamiltonian are the transition rates between different configurations. For the one-dimensional systems defined on a lattice of length L with nearest neighbors integrations the Hamiltonian H has the following general form

$$H = \sum_{k=1}^{L-1} h_{k,k+1} + h_1 + h_L. \tag{2}$$

in which

$$\begin{aligned} h_{k,k+1} &= \mathcal{I}^{\otimes(k-1)} \otimes h \otimes \mathcal{I}^{\otimes(L-k-1)} \\ h_1 &= h^{(l)} \otimes \mathcal{I}^{\otimes(L-1)} \\ h_L &= \mathcal{I}^{\otimes(L-1)} \otimes h^{(r)} \end{aligned} \tag{3}$$

For two-states systems $\mathcal{I}$ is a 2×2 identity matrix and h is a 4×4 matrix for the bulk interactions. The most general form for the bulk Hamiltonian of a

two-states system in the basis $(00, 01, 10, 11)$ is given by

$$h = \begin{pmatrix} -\omega_{11} & \omega_{12} & \omega_{13} & \omega_{14} \\ \omega_{21} & -\omega_{22} & \omega_{23} & \omega_{24} \\ \omega_{31} & \omega_{32} & -\omega_{33} & \omega_{34} \\ \omega_{41} & \omega_{42} & \omega_{43} & -\omega_{44} \end{pmatrix}$$

and for the boundaries by

$$h^{(l)} = \begin{pmatrix} -\alpha & \gamma \\ \alpha & -\gamma \end{pmatrix}, \; h^{(r)} = \begin{pmatrix} -\delta & \beta \\ \delta & -\beta \end{pmatrix}.$$

In this basis 0 and 1 stand for a vacancy and a particle respectively. Requiring the conservation of probability one should have $\omega_{ii} = \sum_{j \neq i} \omega_{ji}$. In the most general form the interaction rates are

$$\begin{array}{ll} \text{Diffusion of particles} & \omega_{32},\ \omega_{23} \\ \text{Coalescence of particles} & \omega_{34},\ \omega_{24} \\ \text{Branching of particles} & \omega_{43},\ \omega_{42} \\ \text{Death of particles} & \omega_{13},\ \omega_{12} \\ \text{Birth of particles} & \omega_{31},\ \omega_{21} \\ \text{Pair Annihilation and Creation} & \omega_{14},\ \omega_{41} \\ \text{Injection and Extraction of particles at the first site} & \alpha,\ \gamma \\ \text{Injection and Extraction of particles at the last site} & \delta,\ \beta. \end{array} \tag{4}$$

In the stationary state we have $HP^*(\mathcal{C}) = 0$. Let us define the occupation number τ_i where $\tau_i = 0$ if the site i is empty and $\tau_i = 1$ if it is occupied by a particle. According to the MPF the stationary probability distribution $P^*(\{\tau_i\})$ is assumed to be of the form

$$P^*(\{\tau_i\}) = \frac{1}{Z_L} \langle W | \prod_{i=1}^{L} (\tau_i \mathbf{D} + (1 - \tau_i)\mathbf{E}) | V \rangle. \tag{5}$$

The function Z_L in (5) is a normalization factor and can be obtained easily using the normalization condition. The operators $\mathbf{D}$ and $\mathbf{E}$ stand for the presence of a particle and a vacancy at each site. These operators besides the vectors $\langle W|$ and $|V\rangle$ should satisfy the following algebra

$$\begin{array}{l} h\left[\begin{pmatrix} \mathbf{E} \\ \mathbf{D} \end{pmatrix} \otimes \begin{pmatrix} \mathbf{E} \\ \mathbf{D} \end{pmatrix}\right] = \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{D}} \end{pmatrix} \otimes \begin{pmatrix} \mathbf{E} \\ \mathbf{D} \end{pmatrix} - \begin{pmatrix} \mathbf{E} \\ \mathbf{D} \end{pmatrix} \otimes \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{D}} \end{pmatrix}, \\ \langle W | \, h^{(l)} \begin{pmatrix} \mathbf{E} \\ \mathbf{D} \end{pmatrix} = -\langle W | \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{D}} \end{pmatrix}, h^{(r)} \begin{pmatrix} \mathbf{E} \\ \mathbf{D} \end{pmatrix} | V \rangle = \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{D}} \end{pmatrix} | V \rangle. \end{array} \tag{6}$$

The operators $\bar{E}$ and $\bar{D}$ are auxiliary operators and do not enter in the calculation of physical quantities.

For three-states systems the Hamiltonian H is given by (2). In this case $\mathcal{I}$ is a 3×3 identity matrix, h is a 9×9 matrix for the bulk interactions and $h^{(l)}$ ($h^{(r)}$) which stands for the particle input and output from the left (right) boundary is a 3×3 matrix. One should note that by requiring the conservation law for the probabilities the matrix h has only 72 independent elements. The boundary matrices $h^{(l)}$ and $h^{(r)}$ have also 6 independent elements each. Introducing the

basis $(00, 01, 02, 10, 11, 12, 20, 21, 22)$, in which 0, 1 and 2 stand for a hole, a particle of type A and a particle of type B respectively, we have

$$h = \begin{pmatrix} -x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & -x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & -x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & -x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & -x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & -x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & -x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & -x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & -x_{99} \end{pmatrix} \tag{7}$$

for the bulk Hamiltonian and also

$$h^{(l)} = \begin{pmatrix} -(\alpha_1 + \alpha_2) & \gamma_1 & \gamma_2 \\ \alpha_1 & -(\gamma_1 + \alpha_3) & \gamma_3 \\ \alpha_2 & \alpha_3 & -(\gamma_2 + \gamma_3) \end{pmatrix} \tag{8}$$

and

$$h^{(r)} = \begin{pmatrix} -(\delta_1 + \delta_2) & \beta_1 & \beta_2 \\ \delta_1 & -(\beta_1 + \delta_3) & \beta_3 \\ \delta_2 & \delta_3 & -(\beta_2 + \beta_3) \end{pmatrix} \tag{9}$$

for the boundaries. As we mentioned the conservation of probabilities in (7) requires $x_{ii} = \sum_{j \neq i} x_{ji}$ for $i, j = 1, \cdots, 9$. At the left boundary the particles of kind A and B are injected into the system with the rates α_1 and α_2 respectively. These particles can also leave the system from there with the rates γ_1 and γ_2. There are also possibilities for changing the particle types at this boundary. The particles of type A (B) are converted to the particles of type B (A) with rate α_3 (γ_3). The same processes can also take place at the right boundary for the particles of type A and B i.e. the particle injection (with the rates δ_1 and δ_2), the particle extraction (with the rates β_1 and β_2) and also the particle type conversion (with the rates δ_3 and β_3). In the three-states case let us define some new notation. We introduce two occupation numbers, τ_i and θ_i, for each site of the lattice, where $\tau_i = 1$ if the site i is occupied by a particle of type A and 0 otherwise. Similarly, $\theta_i = 1$ if the site i is occupied by a particle of type B and 0 otherwise. As the particles are assumed to be subjected to an excluded-volume interaction, only one of τ_i and θ_i can be nonzero and the configuration of the system $\mathcal{C}$ is uniquely defined by the set of occupation numbers $\{\tau_i, \theta_i\}$. According to the MPF the stationary probability distribution $P^*(\{\tau_i, \theta_i\})$ for an open system is assumed to be of the form

$$P^*(\{\tau_i, \theta_i\}) = \frac{1}{Z_L} \langle W | \prod_{i=1}^{L} (\tau_i \mathbf{A} + \theta_i \mathbf{B} + (1 - \tau_i - \theta_i)\mathbf{E}) | V \rangle. \tag{10}$$

The function Z_L in (10) is again a normalization factor and can be obtained easily using the normalization condition. The operators $\mathbf{E}$, $\mathbf{A}$ and $\mathbf{B}$ stand for the presence of a vacancy, a particle of kind A and a particle of kind B at each site. These operators besides the vectors $\langle W|$ and $|V\rangle$ should satisfy the

following algebra

$$h\left[\begin{pmatrix}\mathbf{E}\\ \mathbf{A}\\ \mathbf{B}\end{pmatrix}\otimes\begin{pmatrix}\mathbf{E}\\ \mathbf{A}\\ \mathbf{B}\end{pmatrix}\right]=\begin{pmatrix}\bar{\mathbf{E}}\\ \bar{\mathbf{A}}\\ \bar{\mathbf{B}}\end{pmatrix}\otimes\begin{pmatrix}\mathbf{E}\\ \mathbf{A}\\ \mathbf{B}\end{pmatrix}-\begin{pmatrix}\mathbf{E}\\ \mathbf{A}\\ \mathbf{B}\end{pmatrix}\otimes\begin{pmatrix}\bar{\mathbf{E}}\\ \bar{\mathbf{A}}\\ \bar{\mathbf{B}}\end{pmatrix},$$
$$\langle W|\,h^{(l)}\begin{pmatrix}\mathbf{E}\\ \mathbf{A}\\ \mathbf{B}\end{pmatrix}=-\langle W|\begin{pmatrix}\bar{\mathbf{E}}\\ \bar{\mathbf{A}}\\ \bar{\mathbf{B}}\end{pmatrix},h^{(r)}\begin{pmatrix}\mathbf{E}\\ \mathbf{A}\\ \mathbf{B}\end{pmatrix}|V\rangle=\begin{pmatrix}\bar{\mathbf{E}}\\ \bar{\mathbf{A}}\\ \bar{\mathbf{B}}\end{pmatrix}|V\rangle. \tag{11}$$

As before the operators $\bar{\mathbf{E}}$, $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ are auxiliary operators and do not enter in the calculation of physical quantities. Using the Hamiltonian of the system given by (7)-(9) and (11) the quadratic algebra associated with the most general three-states reaction-diffusion model can be obtained. In order to calculate the mean values of the physical quantities, such as the mean density of particles at each site, one should either find a matrix representation for the algebra or use the commutation relation of the algebra directly.
In the next section we will review the two-states systems in which the Hamiltonian of the system develop a single product shock measure as a random walk provided that the reaction rates fulfill some constraints.

3 Shocks in two-states systems

In [9] exact traveling wave solutions are obtained for three families of one-dimensional two-species reaction-diffusion models with open boundaries. It has been shown that for these models the stationary probability distribution functions can be written as a linear combination of Bernoulli shock measures defined on a lattice of length L as

$$|k\rangle=\begin{pmatrix}1-\rho_1\\ \rho_1\end{pmatrix}^{\otimes k}\otimes\begin{pmatrix}1-\rho_2\\ \rho_2\end{pmatrix}^{\otimes L-k}\quad,\quad 0\leq k\leq L \tag{12}$$

provided that some constraints on the reaction rates are fulfilled. In (12) ρ_1 and ρ_2 are the densities of particles at the left and the right domains of the shock position k respectively. The time evolution of (12) generated by the Hamiltonian of the above-mentioned models is given by

$$\frac{d}{dt}|k\rangle=d_l|k-1\rangle+d_r|k+1\rangle-(d_l+d_r)|k\rangle\quad,\quad 0<k<L \tag{13}$$

which is a simple single-particle random walk equation for the position of the shock k. The shock positions hops to the left and to the right with the rates d_l and d_r respectively. In the following we briefly review the PASEP, the BCRW and the AKGP and the conditions under which we have an exact traveling shock solution for the model [9]. These models are defined on an integer lattice of length L, each site of the lattice i ($1\leq i\leq L$) is either empty or occupied by at most one particle. By applying the standard MPF and defining the operators $\mathbf{D}$ and $\mathbf{E}$ as the operators associated with the existence of particles and vacancies, one finds the following results for the above mentioned models:
For the PASEP the non-vanishing rates in (4) are ω_{32}, ω_{23}, α, β, γ and δ. Using

(6) the quadratic algebra of the PASEP has now the following quadratic form

$$\begin{array}{l}\omega_{23}\mathbf{DE}-\omega_{32}\mathbf{ED}=(\omega_{23}-\omega_{32})(\mathbf{D}+\mathbf{E})\\ \langle W|(\alpha\mathbf{E}-\gamma\mathbf{D})=\langle W|\\ (\beta\mathbf{D}-\delta\mathbf{E})|V\rangle=|V\rangle.\end{array} \tag{14}$$

The auxiliary operators $\bar{\mathbf{E}}$ and $\bar{\mathbf{D}}$ are real numbers proportional to unity in this case. It is known that the Hamiltonian of the PASEP has an eigenvector with zero eigenvalue which can be written in terms of superposition of single product shock measures. The non-vanishing rates together with these densities should satisfy the following conditions

$$\begin{array}{l}\frac{\rho_2(1-\rho_1)}{\rho_1(1-\rho_2)}=\frac{\omega_{23}}{\omega_{32}}\\ \alpha(1-\rho_1)-\gamma\rho_1=\rho_1(1-\rho_1)(\omega_{23}-\omega_{32})\\ \beta\rho_2-\delta(1-\rho_2)=\rho_2(1-\rho_2)(\omega_{23}-\omega_{32}).\end{array} \tag{15}$$

In the bulk of the lattice the shock position hops to the left with the rate $d_l=(\omega_{23}-\omega_{32})\frac{\rho_1(1-\rho_1)}{\rho_2-\rho_1}$ and to the right with the rate $d_r=(\omega_{23}-\omega_{32})\frac{\rho_2(1-\rho_2)}{\rho_2-\rho_1}$. The shock velocity and the shock diffusion coefficient will then can be read as $v_s=d_r-d_l$ and $D_s=(d_r+d_l)/2$. It has also been shown that under the conditions (15) the algebra (14) has a two-dimensional representation [10]. The n-dimensional representation of (14) describes the stationary distribution of the PASEP with $n-1$ consecutive shocks. It has been shown that an n-dimensional irreducible representation for any finite n exists provided that the following constraint is satisfied by the bulk and the boundary rates

$$(\frac{\omega_{32}}{\omega_{23}})^{1-n}=\kappa_+(\alpha,\gamma)\kappa_+(\beta,\delta) \tag{16}$$

in which we have defined

$$\kappa_+(u,v)=\frac{-u+v+1+\sqrt{(u-v-1)^2+4uv}}{2u}. \tag{17}$$

For the BCRW the non-vanishing parameters in (4) are ω_{24}, ω_{42}, ω_{34}, ω_{43}, ω_{32}, ω_{23}, α, β and γ. The dynamics of a single product shock measure under the Hamiltonian of the BCRW is a random walk provided that

$$\begin{array}{l}\frac{1-\rho_1}{\rho_1}=\frac{\omega_{24}+\omega_{34}}{\omega_{42}+\omega_{43}}\\ \frac{1-\rho_1}{\rho_1}=\frac{\omega_{23}}{\omega_{43}}\\ \gamma=\frac{1-\rho_1}{\rho_1}\alpha+(1-\rho_1)\omega_{32}-\frac{1-\rho_1}{\rho_1}\omega_{43}+\rho_1\omega_{34}\end{array} \tag{18}$$

while the density of the particles at the right hand side of the shock position is zero $\rho_2=0$. The shock position hops to the left and right with the hopping rates $d_l=(1-\rho_1)\omega_{32}+\rho_1\omega_{34}$ and $d_r=\omega_{43}/\rho_1$. The shock drift velocity and diffusion coefficient are then $v_s=d_r-d_l$ and $D_s=(d_r+d_l)/2$. Using the standard MPF the quadratic algebra of the BCRW is obtained to be

$$\begin{array}{l}\bar{\mathbf{E}}\mathbf{E}-\mathbf{E}\bar{\mathbf{E}}=0\\ \omega_{23}\mathbf{DE}+\omega_{24}\mathbf{D}^2-(\omega_{32}+\omega_{42})\mathbf{ED}=\bar{\mathbf{E}}\mathbf{D}-\mathbf{E}\bar{\mathbf{D}}\\ -(\omega_{23}+\omega_{43})\mathbf{DE}+\omega_{34}\mathbf{D}^2+\omega_{32}\mathbf{ED}=\bar{\mathbf{D}}\mathbf{E}-\mathbf{D}\bar{\mathbf{E}}\\ \omega_{43}\mathbf{DE}-(\omega_{24}+\omega_{34})\mathbf{D}^2+\omega_{42}\mathbf{ED}=\bar{\mathbf{D}}\mathbf{D}-\mathbf{D}\bar{\mathbf{D}}\\ \langle W|(\alpha\mathbf{E}-\gamma\mathbf{D})=\langle W|\bar{\mathbf{E}}=-\langle W|\bar{\mathbf{D}}\\ \beta\mathbf{D}|V\rangle=\bar{\mathbf{E}}|V\rangle=-\bar{\mathbf{D}}|V\rangle.\end{array} \tag{19}$$

It has been shown that the steady state probability distribution function of the BCRW defined by (5), can be described by a two-dimensional representation of (19) given that the conditions (18) are fulfilled [10].
For the AKGP, the non-vanishing parameters in (4) are ω_{12}, ω_{13}, ω_{42}, ω_{43}, ω_{32}, α and β. The particles are allowed only to enter the system from the first site with the rate α and leave it from the last site of the lattice with the rate β. In this case we have $\rho_1 = 1$ and $\rho_2 = 0$. There are no additional constraints on the rates for this model. The dynamics of shock measure generated by the Hamiltonian of the system will be a simple random walk on the lattice and the shock position hopping rates are $d_l = \omega_{13}$ and $d_r = \omega_{43}$. The shock drift velocity and diffusion coefficient can now be easily calculated. It has also been shown that the quadratic algebra of the AKGP given by

$$\begin{aligned}
&\omega_{13}\mathbf{DE} + \omega_{12}\mathbf{ED} = \bar{\mathbf{E}}\mathbf{E} - \mathbf{E}\bar{\mathbf{E}} \\
&-(\omega_{12} + \omega_{32} + \omega_{42})\mathbf{ED} = \bar{\mathbf{E}}\mathbf{D} - \mathbf{E}\bar{\mathbf{D}} \\
&-(\omega_{13} + \omega_{43})\mathbf{DE} + \omega_{32}\mathbf{ED} = \bar{\mathbf{D}}\mathbf{E} - \mathbf{D}\bar{\mathbf{E}} \\
&\omega_{43}\mathbf{DE} + \omega_{42}\mathbf{ED} = \bar{\mathbf{D}}\mathbf{D} - \mathbf{D}\bar{\mathbf{D}} \\
&\langle W|\alpha\mathbf{E} = \langle W|\bar{\mathbf{E}} = -\langle W|\bar{\mathbf{D}} \\
&\beta\mathbf{D}|V\rangle = \bar{\mathbf{E}}|V\rangle = -\bar{\mathbf{D}}|V\rangle
\end{aligned} \tag{20}$$

has a two-dimensional representation. The stationary state of the system can be written in terms of superposition of product shock measures (12).
In the following section, by defining the total density of particles, we map the quadratic algebra of our three-states system into (14), (19) and (20). We find the conditions under which such mapping is possible.

4 Mapping of algebras

In what follows we consider the most general three-states system consists of two species of particles and vacancies and look for the constraints under which the steady state probability distribution function of the system can be written in terms of superposition of product shock measures. It turns out that the dynamics of such product shock measure under the Hamiltonian of the system is a random walk under the same constraints. In the most general case in a three-states system, the particles belong to two different types A and B and the vacancies which will be denoted by E. We define the total density of particles as the sum of the densities of A and B particles at each lattice site. We assume that at any given site of the lattice the mean occupation value of the particles of type A is always proportional to the mean occupation value of the particles of type B. From the MPF point of view, this means that the operator $\mathbf{A}$, associated with the presence of an A particle at a given site, should be proportional to the the operator $\mathbf{B}$ which is associated with the presence of a B particle i.e. $\mathbf{A} = \frac{1}{r}\mathbf{B}$ in which r is a real and positive parameter. This implies that the operators $\mathbf{A}$ and $\mathbf{B}$ commute with each other but not necessarily with $\mathbf{E}$ which in turn means the stationary probability for occurrence of any configurations of type $\cdots EEEAAABABABBBABEE\cdots$ does not depend on the exact position of A's and B's in each block surrounded by E's rather to the number of them. We should emphasis that such constraint does not result in a trivial two-states model even at the microscopic level (see [13] for instance). Now by defining the total density of particles operator $\mathbf{D}$ as $\mathbf{D} := \mathbf{A} + \mathbf{B}$ the associated quadratic algebra

of the three-states system (11) can be written in terms of the two operators $\mathbf{D}$ and $\mathbf{E}$ and the auxiliary operators $\bar{\mathbf{E}}$, $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$. We also assume that $\bar{\mathbf{A}} = \frac{1}{r}\bar{\mathbf{B}}$ and define $\bar{\mathbf{D}} := \bar{\mathbf{A}} + \bar{\mathbf{B}}$. Under these assumptions one finds a quadratic algebra in term of the four operators $\mathbf{D}$, $\bar{\mathbf{D}}$, $\mathbf{E}$ and $\bar{\mathbf{E}}$. This algebra can be regarded as an associated quadratic algebra of a two-states system with open boundaries. Now the question is *"what do we get if we try to map this quadratic algebra into the quadratic algebras associated with the PASEP, the BCRW and that of the AKGP?"*. Such mapping will indeed impose some constraints on the reaction rates of the three-states system. The first result is that the quadratic algebra of the three-states system can have two-dimensional representations by imposing extra constraints. This means that the stationary distribution of the three-states system, when it is described in terms of the total density of particles and the density of vacancies, can be written in terms of superposition of single product shock measures. Moreover, if the total density of the particles has a step-function structure, it will evolve similar to a random walker in continues time under the Hamiltonian of the system. In the following, we map the algebra of the three-states system in terms of $\mathbf{D}$ and $\mathbf{E}$ into (14), (19) and (20) and investigate the outcomes. As we mentioned, this will impose some conditions on the reaction rates of our three-states system i.e. x_{ij}'s introduced in (7) and also the boundary rates in (8) and (9). The constraints (15) and (18) in the case of mapping into the PASEP's and the BCRW's algebra will then be applied to the total density of particles and the newly defined boundary rates. These constraints will guarantee that the dynamics of (12) will be a simple random walk and that the steady state of the system can be written in terms of superposition of these shocks.

4.1 Mapping into the PASEP's algebra

Here we introduce the conditions under which the quadratic algebra of the three-states system can be mapped into (14). One should note that the quadratic algebra of the PASEP does not contain any quadratic terms of types $\mathbf{E}^2$ or $\mathbf{D}^2$. Since we have defined $\mathbf{A} = \frac{1}{1+r}\mathbf{D}$ and $\mathbf{B} = \frac{r}{1+r}\mathbf{D}$; therefore, the quadratic algebra of the three-states system should not contain the quadratic terms of types $\mathbf{A}^2$, $\mathbf{B}^2$ or $\mathbf{E}^2$. This imposes some constraints on the matrix elements of (7). On the other hand, in order to have a quadratic term of type $\mathbf{DE}$ one should only have the quadratic terms of types $\mathbf{AE}$ and $\mathbf{BE}$. All other combinations of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ should also be eliminated in the quadratic algebra of the three-sates system. By applying these constraints we have found that the most general Hamiltonian of the three-states system (7) should have the following form in order to be mapped into (14)

$$h = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -x_{22} & x_{23} & x_{24} & 0 & 0 & x_{27} & 0 & 0 \\ 0 & x_{32} & -x_{33} & x_{34} & 0 & 0 & x_{37} & 0 & 0 \\ 0 & x_{42} & x_{43} & -x_{44} & 0 & 0 & x_{47} & 0 & 0 \\ 0 & 0 & 0 & 0 & -x_{55} & x_{56} & 0 & x_{58} & x_{59} \\ 0 & 0 & 0 & 0 & x_{65} & -x_{66} & 0 & x_{68} & x_{69} \\ 0 & x_{72} & x_{73} & x_{74} & 0 & 0 & -x_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{85} & x_{86} & 0 & -x_{88} & x_{89} \\ 0 & 0 & 0 & 0 & x_{95} & x_{96} & 0 & x_{98} & -x_{99} \end{pmatrix} \tag{21}$$

in which $x_{ii} = \sum_{j \neq i} x_{ji}$ for $i = 2, \cdots, 9$. All of the boundary rates in (8) and (9) are non-zero in this case. We assume that the auxiliary operators $\bar{\mathbf{D}}$ and $\bar{\mathbf{E}}$ are proportional to real numbers $\bar{\mathbf{d}}$ and $\bar{\mathbf{e}}$. In this case one finds that $\bar{\mathbf{d}} = -\bar{\mathbf{e}}$. By defining $\bar{\mathbf{e}} := \omega_{32} - \omega_{23}$ one can see that the algebra of the three-states system can be mapped into (14) provided that

$$\begin{aligned} &rx_{68} + r^2 x_{69} + x_{65} = rx_{66} \\ &rx_{86} + r^2 x_{89} + x_{85} = rx_{88} \\ &r(x_{96} + x_{98}) + x_{95} = r^2 x_{99}. \end{aligned} \tag{22}$$

One should also define the bulk parameters as follows

$$\begin{aligned} \omega_{23} &:= x_{24} + rx_{27} = x_{37} + \tfrac{1}{r} x_{34} \\ &= x_{44} - rx_{47} = x_{77} - \tfrac{1}{r} x_{74} \\ \omega_{32} &:= x_{42} + rx_{43} = x_{73} + \tfrac{1}{r} x_{72} \\ &= x_{22} - rx_{23} = x_{33} - \tfrac{1}{r} x_{32} \end{aligned} \tag{23}$$

and the boundary rates as

$$\begin{aligned} &\alpha := \tfrac{(1+r)\alpha_1}{\omega_{23} - \omega_{32}} = \tfrac{\alpha_2 (1+r)}{r(\omega_{23} - \omega_{32})} \\ &\beta := \tfrac{\beta_1 + \delta_3 - r\beta_3}{\omega_{23} - \omega_{32}} = \tfrac{r(\beta_2 + \beta_3) - \delta_3}{r(\omega_{23} - \omega_{32})} \\ &\gamma := \tfrac{r(\gamma_2 + \gamma_3) - \alpha_3}{r(\omega_{23} - \omega_{32})} = \tfrac{\gamma_1 + \alpha_3 - r\gamma_3}{\omega_{23} - \omega_{32}} \\ &\delta := \tfrac{(1+r)\delta_1}{\omega_{23} - \omega_{32}} = \tfrac{(1+r)\delta_2}{r(\omega_{23} - \omega_{32})}. \end{aligned} \tag{24}$$

The conditions in (15) should now be applied to the total density of the particles and the new boundary parameters defined in (24).
In [13] the authors have introduced a three-states system with a Hamiltonian similar to (21) and shown that the dynamics of a single product shock measure under this Hamiltonian will be a random walk provided that the constraints similar to (22)-(24) are fulfilled. It should be mentioned that since (14) has also finite-dimensional representations, multiple shock structures may evolve in this system under the condition (16). Mapping the quadratic algebra of the three-states system into the PASEP's algebra not only confirms the results obtained in [13] but also prove the possibility of evolving multiple shocks in the same system which has not been studied before. One should also note that the models studied in [14] and [15] are special cases of the system studied here.

4.2 Mapping into the BCRW's algebra

In order to map the quadratic algebra of the three-states system into (19) we define the total density operator $\mathbf{D}$ and rewrite the algebra in terms of the two operators $\mathbf{D}$ and $\mathbf{E}$ as before. Nevertheless, the auxiliary operators are not assumed to be real numbers is this case. One should note that the quadratic algebra of the BCRW does not contain any quadratic term of type $\mathbf{E}^2$. This means that all of the entries in the first column of (11) should be equal to zero. All other combinations of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ are allowed; therefore, the most general

Hamiltonian should have the following form

$$h = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ 0 & x_{32} & -x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ 0 & x_{42} & x_{43} & -x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ 0 & x_{52} & x_{53} & x_{54} & -x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ 0 & x_{62} & x_{63} & x_{64} & x_{65} & -x_{66} & x_{67} & x_{68} & x_{69} \\ 0 & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & -x_{77} & x_{78} & x_{79} \\ 0 & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & -x_{88} & x_{89} \\ 0 & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & -x_{99} \end{pmatrix} \tag{25}$$

in which, for the conservation of the probability, x_{ii}'s are defined so that the sum of the elements of each column is zero. The reason that the entries of the first row in (25) are taken to be equal to zero comes from the fact that it gives a linear combination of operators with positive coefficients equal to zero. This cannot be true unless the coefficients (reaction rates on the first row) are zero. We now can map the quadratic algebra of the three-states system into (19) provided that the nonzero parameters of the BCRW's algebra are defined as follows

$$\begin{aligned} \omega_{23} &:= x_{24} + r x_{27} = \tfrac{1}{r}(x_{34} + r x_{37}) \\ \omega_{24} &:= \tfrac{1}{1+r}(x_{25} + r(x_{26} + x_{28}) + r^2 x_{29}) \\ &= \tfrac{1}{r(1+r)}(x_{35} + r(x_{36} + x_{38}) + r^2 x_{39}) \\ \omega_{32} &:= x_{42} + r x_{43} = \tfrac{1}{r}(x_{72} + r x_{73}) \\ \omega_{34} &:= \tfrac{1}{1+r}(x_{45} + r(x_{46} + x_{48}) + r^2 x_{49}) \\ &= \tfrac{1}{r(1+r)}(x_{75} + r(x_{76} + x_{78}) + r^2 x_{79}) \\ \omega_{42} &:= (1+r)(x_{52} + r x_{53}) = \tfrac{1+r}{r}(x_{62} + r x_{63}) \\ &= \tfrac{1+r}{r}(x_{82} + r x_{83}) = \tfrac{1+r}{r^2}(x_{92} + r x_{93}) \\ \omega_{43} &:= (1+r)(x_{54} + r x_{57}) = \tfrac{1+r}{r}(x_{64} + r x_{67}) \\ &= \tfrac{1+r}{r}(x_{84} + r x_{87}) = \tfrac{1+r}{r^2}(x_{94} + r x_{97}). \end{aligned} \tag{26}$$

There are also some extra constraints that should be fulfilled

$$\begin{aligned} x_{22} - r x_{23} &= \tfrac{1}{r}(r x_{33} - x_{32}) \\ x_{44} - r x_{47} &= \tfrac{1}{r}(r x_{77} - x_{74}) \\ x_{55} - r(x_{56} + x_{58}) - r^2 x_{59} &= \tfrac{1}{r}(-x_{65} + r(x_{66} - x_{68}) - r^2 x_{69}) \\ &= \tfrac{1}{r}(-x_{85} + r(x_{88} - x_{86}) - r^2 x_{89}) \\ &= \tfrac{1}{r^2}(-x_{95} - r(x_{96} + x_{98}) + r^2 x_{99}). \end{aligned} \tag{27}$$

The boundary rates in this case should be defined as follows

$$\begin{aligned} &\alpha := \alpha_1(1+r) = \alpha_1 + \alpha_2 \\ &\beta := \beta_1 - r\beta_3 + \delta_3 = \tfrac{1}{1+r}(\beta_1 + r\beta_2) \\ &\gamma := \gamma_1 + \alpha_3 - r\gamma_3 = \tfrac{1}{1+r}(\gamma_1 + r\gamma_2) \\ &\delta := \delta_1 = \delta_2 = 0. \end{aligned} \tag{28}$$

The last constraint in (28) indicates that there is no particle injection from the right boundary. The conditions given by (18) should now be applied to the total density of particles and newly defined parameters in (26) and (28). These conditions and definitions are enough for writing the steady state probability distribution function of the model in terms of two-dimensional representation of the algebra.

4.3 Mapping into the AKGP's algebra

In the case of mapping the quadratic algebra of our three-states system into the AKGP's algebra, the Hamiltonian of the three-states system (7) should have the following form

$$h = \begin{pmatrix} 0 & x_{12} & x_{13} & x_{14} & 0 & 0 & x_{17} & 0 & 0 \\ 0 & -x_{22} & x_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{32} & -x_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{42} & x_{43} & -x_{44} & 0 & 0 & x_{47} & 0 & 0 \\ 0 & x_{52} & x_{53} & x_{54} & 0 & 0 & x_{57} & 0 & 0 \\ 0 & x_{62} & x_{63} & x_{64} & 0 & 0 & x_{67} & 0 & 0 \\ 0 & x_{72} & x_{73} & x_{74} & 0 & 0 & -x_{77} & 0 & 0 \\ 0 & x_{82} & x_{83} & x_{84} & 0 & 0 & x_{87} & 0 & 0 \\ 0 & x_{92} & x_{93} & x_{94} & 0 & 0 & x_{97} & 0 & 0 \end{pmatrix}. \tag{29}$$

Since (20) does not contain any quadratic terms of types $\mathbf{D}^2$ and $\mathbf{E}^2$, the quadratic algebra of the three-states system should not contain the terms $\mathbf{A}^2$, $\mathbf{B}^2$, $\mathbf{AB}$, $\mathbf{BA}$ and $\mathbf{E}^2$; therefore, the first, the fifth, the sixth and the last two columns of (29) are chosen to be zero; however, all other combinations of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ are allowed in this case. In order to map the quadratic algebra of the three-states model in terms of the operators $\mathbf{D}$, $\mathbf{E}$, $\bar{\mathbf{D}}$ and $\bar{\mathbf{E}}$ into the AKGP's algebra given by (20) the nonzero parameters in (29) should be related to the nonzero parameters of the AKGP as follows

$$\begin{array}{rcl} \omega_{12} & := & \frac{1}{1+r}(x_{12} + r x_{13}) \\ \omega_{13} & := & \frac{1}{1+r}(x_{14} + r x_{17}) \\ \omega_{32} & := & x_{42} + r x_{43} \\ & = & \frac{1}{r}(x_{72} + r x_{73}) \\ \omega_{42} & := & (1+r)(x_{52} + r x_{53}) \\ & = & \frac{1+r}{r}(x_{62} + r x_{63}) \\ & = & \frac{1+r}{r}(x_{82} + r x_{83}) \\ & = & \frac{1+r}{r^2}(x_{92} + r x_{93}) \\ \omega_{43} & := & (1+r)(x_{54} + r x_{57}) \\ & = & \frac{1+r}{r}(x_{64} + r x_{67}) \\ & = & \frac{1+r}{r}(x_{84} + r x_{87}) \\ & = & \frac{1+r}{r^2}(x_{94} + r x_{97}) \end{array} \tag{30}$$

besides the following constraints

$$\begin{array}{l} x_{22} - r x_{23} = x_{33} - \frac{x_{32}}{r} \\ x_{44} - r x_{47} = x_{77} - \frac{x_{74}}{r} \end{array} \tag{31}$$

in which x_{ii}'s for $i = 2, 3, 4$ and 7 are defined according to the conservation of probabilities. For the boundary rates one should have $\alpha_3 = r\gamma_3$. On the other hand we should define

$$\begin{array}{l} \alpha := (1+r)\alpha_1 = \alpha_1 + \alpha_2 \\ \beta := \beta_1 - r\beta_3 + \delta_3 = \frac{1}{1+r}(\beta_1 + r\beta_2) \\ \gamma := \gamma_1 = \gamma_2 = 0 \\ \delta := \delta_1 = \delta_2 = 0. \end{array} \tag{32}$$

These conditions are enough in order to write the steady state of the three-states system in terms of two-dimensional matrices which satisfy (20).

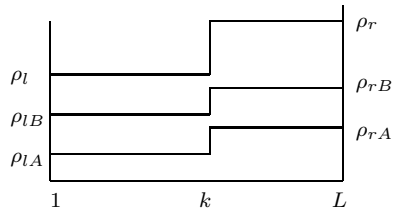

Figure 1: Sketch of a single shock at sites k on a lattice of length L. The density profile of A particles is always assumed to be proportional to the density profile of B particles both on the left hand and the right hand sides of the shock position.

5 Dynamics of shock

Let us represent a product shock measure with the shock position on the site k for our three-states system as follows

$$|k\rangle = \begin{pmatrix} 1-\rho_{lA}-\rho_{lB} \\ \rho_{lA} \\ \rho_{lB} \end{pmatrix}^{\otimes k} \otimes \begin{pmatrix} 1-\rho_{rA}-\rho_{rB} \\ \rho_{rA} \\ \rho_{rB} \end{pmatrix}^{\otimes L-k} \tag{33}$$

in which ρ_{lA} (ρ_{rA}) and ρ_{lB} (ρ_{rB}) are the densities of A and B particles on the left (right) hand side of the shock position respectively. According to our assumption that the density of A particles is always proportional to that of B particles, one should have $\rho_{lB} = r\rho_{lA}$ and $\rho_{rB} = r\rho_{rA}$. Now defining the total density of particles on the sides of the shock position as ρ_l and ρ_r, we have $\rho_l = (1+r)\rho_{lA}$ and $\rho_r = (1+r)\rho_{rA}$. Sketch of such shock measure can be seen in figure 1. In the first case (mapping the quadratic algebra of the three-states system into that of the PASEP) the shock (33) has a random walk dynamics under (21) provided that the constraints (22)-(24) are satisfied. In this case we obtain

$$\rho_l = \frac{1}{1+\kappa_+(\alpha,\gamma)} \tag{34}$$

$$\rho_r = \frac{\kappa_+(\beta,\delta)}{1+\kappa_+(\beta,\delta)} \tag{35}$$

in which we have used (17) and the boundary rates α, β, γ and δ are defined according to (24). These boundary rates besides the total density of particles satisfy the following constraint

$$\begin{array}{l} \frac{\rho_r(1-\rho_l)}{\rho_l(1-\rho_r)} = \frac{\omega_{23}}{\omega_{32}} \\ \alpha(1-\rho_l) - \gamma\rho_l = \rho_l(1-\rho_l)(\omega_{23}-\omega_{32}) \\ \beta\rho_r - \delta(1-\rho_r) = \rho_r(1-\rho_2)(\omega_{23}-\omega_{32}). \end{array} \tag{36}$$

The shock position hops to the left and the right with the rates $\frac{1-\rho_l}{1-\rho_r}w_{32}$ and $\frac{\rho_r}{\rho_l}w_{32}$ respectively.
In the second case (mapping the quadratic algebra of the three-states system to that of the BCRW) the shock (33) performs a random walk again under (25)

provided that (26)-(28) are fulfilled. The total density of the particle on the right hand side of the shock is zero. The shock position in this case hops to the left and to the right with the rates $(1-\rho_l)\omega_{32}+\rho_l\omega_{34}$ and ω_{43}/ρ_l respectively in which ω_{32}, ω_{34} and ω_{43} are defined in (26). The total density of the particles on the left hand side of the shock ρ_l, the boundary parameters α and δ, besides the non-zero parameters defined in (26) should now satisfy the following constraints

$$\begin{array}{l} \frac{1-\rho_l}{\rho_l}=\frac{\omega_{24}+\omega_{34}}{\omega_{42}+\omega_{43}} \\ \frac{1-\rho_l}{\rho_l}=\frac{\omega_{23}}{\omega_{43}} \\ \gamma=\frac{1-\rho_l}{\rho_1}\alpha+(1-\rho_l)\omega_{32}-\frac{1-\rho_l}{\rho_l}\omega_{43}+\rho_l\omega_{34}. \end{array} \tag{37}$$

In the third case of mapping the quadratic algebra of the three-states system to the algebra of the AKGP the shock (33) performs a random walk under the Hamiltonian (29). In this case the total density of the particles of the left hand side of the shock position is equal to unity while it is zero on the right hand side of the shock position. The shock position hops to its leftmost (rightmost) site with the rate $w_{13}(w_{43})$. There is no need to introduce any extra constraints more than those in (31) in this case.

6 Concluding remarks

In this paper we have considered the most general Hamiltonian for the Markovian three-states systems (two species of particles besides the vacancies) with open boundaries. From the MPF point of view, the quadratic algebra of this system is generated by three operators **A**, **B**, and **E** associated with the existence of two species of particles and the vacancies. We have then assumed that the density of particles are proportional to each other. By defining the total density of the particles as sum of the densities of the particles, this algebra transforms into a quadratic algebra in terms of the total density operator **D** and **E**. Now this algebra can be treated as the quadratic algebra of a two-states system. We have shown that under certain conditions this algebra can be mapped to the quadratic algebras of the PASEP, the BCRW and that of the AKGP. Such mapping has two immediate results: The stationary state of our three-states system can be written in terms of superposition of product shock measures and that the dynamics of a single product shock measure given by the Hamiltonian of the system is simply a random walk.

One can simply check that the conditions which are necessary for mapping the algebra of the three-states system into the PASEP's algebra, are exactly those introduced in [13] but obtained from different approach. In [13] the authors have introduced a single product shock measure and found the conditions under which it has a simple random walk dynamics under the Hamiltonian (14). In this paper we have shown that (14) is not the only way one can define the Hamiltonian in order to have such property. There are actually two other ways which have been introduced and studied here. In the same reference the authors have only studied the dynamics of a single shock. We have shown that, at least for the case of mapping the algebra of the three-states system into the PASEP's algebra, multiple shocks can evolve in the system.

One should note that the procedure introduced in this paper is not the only way one can write the quadratic algebra of a three-states system in terms of the

two operators **D** and **E**. It is worth studying the case where the operators **A** and **B** are not proportional but related to each other. Our approach can also be generalized to the systems with more than three states at each lattice site. This is under our investigations [16].

References

[1] B. Schmittmann and R. K. P. Zia, *Phase Transitions and Critical Phenomena*, Vol 17, C. Domb and J. Lebowitz eds. (Academic, London, 1994)

[2] G. M. Schütz *Phase Transitions and Critical Phenomena*, Vol 19, C. Domb and J. Lebowitz eds. (Academic, London, 2001)

[3] B. Derrida, M.R. Evans, V. Hakim and V. Pasquier *J. Phys. A: Math. Gen.* A **26** 1493 (1993)

[4] P. A. Ferrari *Prob. Theor. and Relat. Fields* **91** 81 (1992)

[5] P. A. Ferrari and L. R. G. Fontes *Prob. Theor. and Relat. Fields* **99** 305 (1994)

[6] B. Derrida, M. R. Evans and K. Mallick *J. Stat. Phys.* **79** 833 (1995)

[7] V. Belitsky and G. M. Schütz *El. J. Prob.* **7** Paper No.11 1 (2002)

[8] C. Pigorsch C and G.M. Schütz *J. Phys. A* **33** 7919 (2000)

[9] K. Krebs, F. H. Jafarpour, and G. M. Schütz *New Journal of Physics* **5** 145.1-145.14 (2003)

[10] F. H. Jafarpour Physica A **339** 369 (2004)

[11] F.H.L. Essler and V. Rittenberg *J. Phys. A: Math. Gen.* A **29** 3375 (1996)

[12] K. Mallick and S. Sandow *J. Phys. A: Math. Gen.* A **30** 4513 (1997)

[13] F. Tabatabaei, G.M. Schütz *Phys. Rev.* E **74** 051108 (2006)

[14] A. Rákos and G. M. Shcütz *J. Stat. Phys.* **117** 55 (2004)

[15] F. H. Jafarpour Physica A **358** 413 (2005)

[16] F. H. Jafarpour and S. R. Masharian, In progress.